Honors Algebra 2 – Unit 5 Day #2
Homework – Inverse Functions

Name Key

1. Solve the equation $3 = 8^x$ to the nearest hundredths.

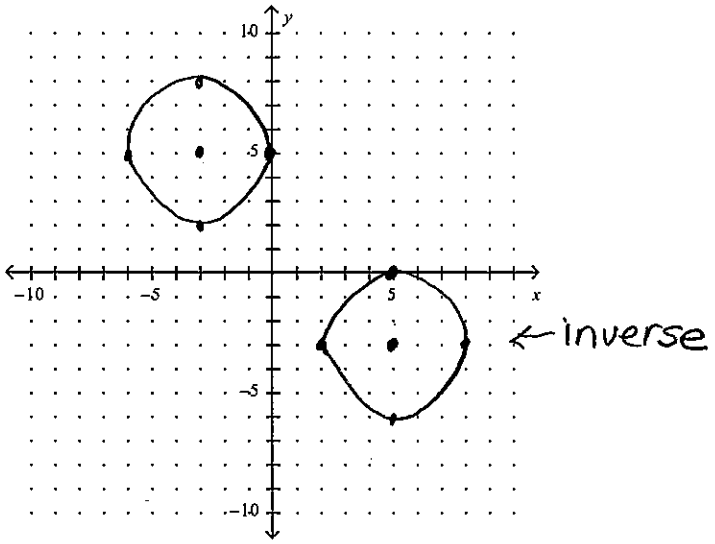
Graph $y = 8^x$ and $y = 3$ } Calculate Intersection $x = 0.53$

2. Write the equation of a circle that has a center at $(-3,5)$ and a point on the circle at $(0,5)$. Graph the circle and its inverse.

$(x+3)^2 + (y-5)^2 = 9$

inverse

$(x-5)^2 + (y+3)^2 = 9$



3. Dana's mother gave her \$175 for her sixteenth birthday, requiring her to put the money into her savings account until her eighteenth birthday. Dana already has \$237.54 in her account, which pays 3.25% interest, compounded quarterly. How much will Dana have in the account on her eighteenth birthday assuming she makes no deposits or withdrawals?

237.54
175.00
\$412.54

$A = 412.54\left(1 + \frac{.0325}{4}\right)^{4 \cdot 2}$

$A = \$440.13$

4. Let $f(x) = \frac{4}{5}x + 1$ and $g(x) = \frac{5}{4}x - \frac{5}{4}$. Are $f(x)$ and $g(x)$ inverses of one another? Prove your answer using composites.

$f(g(x)) = \frac{4}{5}\left(\frac{5}{4}x - \frac{5}{4}\right) + 1$

$= x - 1 + 1$

$= x$

$g(f(x)) = \frac{5}{4}\left(\frac{4}{5}x + 1\right) - \frac{5}{4}$

$= x + \frac{5}{4} - \frac{5}{4}$

$= x$

$f(x)$ and $g(x)$ are inverses because

$f(g(x)) = g(f(x)) = x$

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5. If $2^{x+4} = 4^{3x-1}$, what is the value of x?

$2^{x+4} = 2^{2(3x-1)}$

$x+4 = 6x-2$

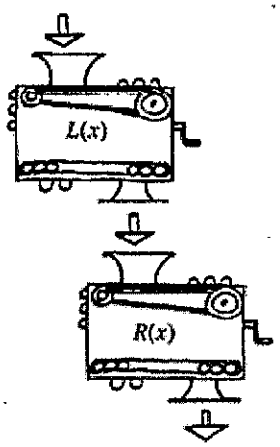
$6 = 5x$

$x = \frac{6}{5}$

6. Laura and Robert each have their own personal function machine. Laura's machine, $L(x)$, squares the input and then subtracts one. Robert's function machine, $R(x)$, adds 2 to the input and then multiples the result by three.

a. Write the equations that represent $L(x)$ and $R(x)$.

$L(x) = x^2 - 1$ $\quad R(x) = 3(x+2)$



b. Laura and Robert decide to connect their two machines so that Laura's output becomes Robert's input. If 3 is the initial input, what is the final output?

$R(L(3)) = R(8) = 30$

c. What if they switch the order of the machines? Would they get the same result? NO Justify your answer.

$L(R(3)) = L(15) = 224$

Switching the order only works if $L(x)$ & $R(x)$ are inverses

Find the inverse of each of the following functions algebraically.

7. $y = 16(x-8)^2 + 3$

$16(y-8)^2 + 3 = x$

$(y-8)^2 = \frac{x-3}{16}$

$y-8 = \pm\frac{\sqrt{x-3}}{4}$

$y = 8 \pm \frac{\sqrt{x-3}}{4}$

8. $y = \left(\frac{1}{4}x + 6\right)^3$

$\left(\frac{1}{4}y+6\right)^3 = x$

$\frac{1}{4}y+6 = \sqrt[3]{x}$

$y = 4(\sqrt[3]{x}-6)$

$= 4\sqrt[3]{x} - 24$

9. $y = \sqrt{5x-6} + 1$

$\sqrt{5y-6} + 1 = x$

$5y-6 = (x-1)^2$

$y = \frac{(x-1)^2}{5} + \frac{6}{5}$

10. $y = \frac{2}{x+1} - 7$

$\frac{2}{y+1} - 7 = x$

$\frac{2}{y+1} = \frac{x+7}{1}\cdot\frac{(y+1)}{(y+1)}$

$2 = (x+7)(y+1)$

$\frac{2}{x+7} = y+1$ $\quad y = \frac{2}{x+7} - 1$

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11. Let $f(x) = \frac{2-x}{3}$. Find $f(f(f(x)))$

$f(f(x)) = \frac{2-\frac{2-x}{3}}{3}$

$= \frac{\left(\frac{6-2+x}{3}\right)\cdot 1/3}{3\cdot 1/3}$

$= \frac{4+x}{9}$

$f\left(\frac{4-x}{9}\right) = \frac{2-\left(\frac{4+x}{9}\right)}{3}$

$= \frac{\frac{18-4-x}{9}\cdot 1/3}{3\cdot 1/3}$

$= \frac{14-x}{27}$

For #12 - 14, let $f(x) = \frac{1}{4x^2-9}$ and $g(x) = -2x - 4$

12. a. Evaluate $f(g(3))$

$g(3) = -6-4 = -10$

$f(-10) = \frac{1}{4(100)-9}$

$f(g(3)) = \frac{1}{391}$

b. Evaluate $g\left(f\left(\frac{-1}{2}\right)\right)$

$f\left(-\frac{1}{2}\right) = \frac{1}{4\left(\frac{1}{4}\right)-9} = \frac{-1}{8}$

$g\left(-\frac{1}{8}\right) = -2\left(-\frac{1}{8}\right)-4 = \frac{1}{4}-4$

$g\left(f\left(-\frac{1}{2}\right)\right) = -\frac{15}{4}$

13. Find an expression for $f(g(x))$.

$= \frac{1}{4(-2x-4)^2-9}$

$= \frac{1}{4(4x^2+16x+16)-9}$

$= \frac{1}{16x^2+64x+55}$

14. State the domain restrictions for $f(g(x))$.

$4(-2x-4)^2-9 \neq 0$

$(-2x-4)^2 \neq \frac{9}{4}$

$-2x-4 \neq \frac{3}{2}$ $\quad -2x-4 \neq -\frac{3}{2}$

$-2x \neq \frac{11}{2}$ $\quad -2x \neq \frac{5}{2}$

$x \neq -\frac{11}{4}$ $\quad x \neq -\frac{5}{4}$

15. Write the equation of exponential function whose graph contains the points (2, 24) and (4, 432).

$432 = ab^4$

$\div \; 24 = ab^2$

$18 = b^2$

$3\sqrt{2} = b$

$24 = a(3\sqrt{2})^2$

$24 = 18a$

$\frac{4}{3} = a$

$y = \frac{4}{3}(3\sqrt{2})^x$

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16. Caesium – 137 is a radioactive isotope formed by a nuclear reactor during its operation. During the Chernobyl disaster in 1986, 500 grams were found in a closed lake in the Ukraine. The half-life period of Caesium – 137 is 30.17 years.

a. How many grams are still remaining today?

$A = 500\left(\frac{1}{2}\right)^{t/30.17}$

2018
$y = 500\left(\frac{1}{2}\right)^{32/30.17}$
$= 239.71 \text{ grams}$

2019
$y = 500\left(\frac{1}{2}\right)^{33/30.17}$
$= 234.26 \text{ grams}$

b. How many years will it take for there to be less than 5 grams? Write down the steps you used to find the answer.

$y = 5$
$y = 500(.5)^{\frac{x}{30.17}}$

Graph
Calculate
Intersection

It will take 201 years for there to be less than 5 grams.

Simplify the following expressions.

17. $\left(729^{\frac{4}{3}}\right)^{\frac{5}{8}}$

$= (6561)^{\frac{5}{8}}$
$= 243$

18. $\left(\frac{2187}{16{,}384}\right)^{\frac{4}{7}} \cdot \left(\frac{4096}{5832}\right)^{\frac{2}{3}}$

$= \frac{81}{256} \cdot \frac{256}{324}$
$= \frac{81}{324}$
$= \frac{1}{4}$

19. $\left(x^{\frac{3}{4}} \div x^{\frac{7}{8}}\right) \cdot x^{\frac{-1}{6}}$

$= \left(x^{\frac{6}{8}-\frac{7}{8}}\right) \cdot x^{-\frac{1}{6}}$
$= x^{-\frac{1}{8}} \cdot x^{-\frac{1}{6}}$
$= x^{-\frac{3}{24}+\frac{-4}{24}}$
$= \frac{1}{x^{\frac{7}{24}}}$

20. $\left(\left(\frac{x^{\frac{1}{4}}y^{-2}}{x^{-\frac{1}{2}}y^{\frac{1}{4}}}\right)^{\frac{-2}{3}} \cdot x^{-3}\right)^{-2}$

$= \left(\frac{x^{\frac{3}{4}}}{y^{\frac{3}{4}}}\right)^{-\frac{2}{3}}$

$= \left(\frac{y^{\frac{3}{2}}}{x^{\frac{1}{2}}} \cdot x^{-3}\right)^{6}$
$= \frac{y^{-3}}{x^{-1}} \cdot x$
$= \frac{x \cdot x^{6}}{y^{3}} = \frac{x^{7}}{y^{3}}$